Exercise: ship tracking from bearings-only

We will consider three different filtering approaches (Extended Kalman filter, Particle filter, Ensemble Kalman filter) for the task of tracking a ship on the surface using two bearings-only measurements.

There are two sensors measuring angles to a ship. One sensor is located in east,north coordinate $(0, 0)$, the other at $(40, 40)$. Units are in km. See Figure 1 for a map view of this situation with the initial measurement and an example of a realized ship trajectory.

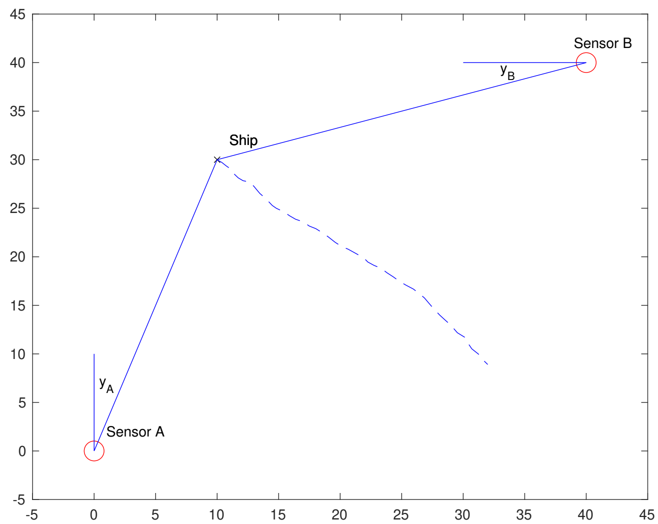

Figure 1: Sensor A and B in bearing-only tracking.

The state space model is defined as follows: A surface vessel has state $\boldsymbol{x}_t = (E_t, N_t, v_t, u_t)^t$ [km], where (E_t, N_t) [km/h] is the east and north position at time t, while (v_t, u_t) is the associated velocity vector at time t. Initially, $\boldsymbol{x}_1 \sim N(\boldsymbol{\mu}_1, \boldsymbol{\Sigma}_1)$, where the mean is $\boldsymbol{\mu}_1 = (10, 30, 15, -15)^t$ and $\boldsymbol{\Sigma}_1 = \text{diag}(5^2, 5^2, 3^2, 3^2)^t$. The dynamic process model is defined by

$$\boldsymbol{x}_{t+1} = \boldsymbol{A}\boldsymbol{x}_t + \boldsymbol{\epsilon}_{t+1}, \quad \boldsymbol{A} = \begin{bmatrix} 1 & 0 & \delta & 0 \\ 0 & 1 & 0 & \delta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ \boldsymbol{\epsilon}_{t+1} \sim N(\boldsymbol{0}, \text{diag}[0.1^2, 0.1^2, 0.7^2, 0.7^2]),$$

for $t = 1, \ldots, T-1$, and with $\delta = 1/60$ which means sampling interval of every minute. The measurements at sensors A and B; $\boldsymbol{y}_t = (y_{A,t}, y_{B,t})^t$, $t = 1, \ldots, T$, are modeled as conditionally independent and with Gaussian noise so that

$$\boldsymbol{y}_t = \begin{bmatrix} \arctan(E_t/N_t) \\ \arctan[(40-N_t)/(40-E_t)] \end{bmatrix} + N(\mathbf{0}, \text{diag}[0.087^2, 0.087^2]),$$

where the inverse tangent function is defined as in Figure 1, assuming we know that the ship is entering both sensors' field of interest. The noise level corresponds to a standard deviation of 5 degrees in the angle.

The angle measurements at sensor A and B can be downloaded from `https://folk.ntnu.no/joeid/MA8702/sensorA.txt` and `https://folk.ntnu.no/joeid/MA8702/sensorB.txt`. These measurements are displayed over $T = 50$ time steps in Figure 2.

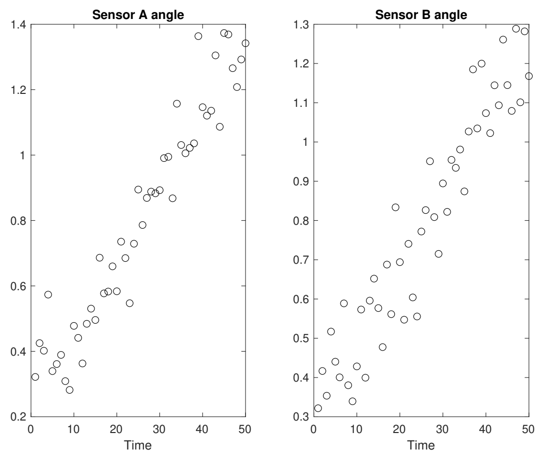

Figure 2: Sensor A and B measurements.

TASKS

The goal is to analyze the filtering density $p(\boldsymbol{x}_t|\boldsymbol{y}_1, \ldots, \boldsymbol{y}_t)$, $t = 1, \ldots, T$.

- Implement an extended Kalman filter solution to find an approximate solution. This means linearizing the measurement equation around the predicted state. Plot the predicted solution in a map, along with uncertainty bounds.
- Implement a standard particle filter with $B = 1000$ particles, using the state Markovian process model as proposal at each time step. Apply resampling in the particle filter. Plot the predicted solution in a map, along with uncertainty bounds.
- Implement a standard ensemble Kalman filter with $B = 1000$ ensemble members. Plot the predicted solution in a map, along with uncertainty bounds.