Value of Information Analysis in Spatial Models

Jo Eidsvik

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My background:

Professor of Statistics at NTNU in Trondheim, NORWAY.

Education:

- MSc in Applied Mathematics, Univ of Oslo
- PhD in Statistics, NTNU

Work experience:

- Norwegian Defense Research Establishment
- Statoil

Research interests:

- Spatial statistics, spatio-temporal statistics,
- Computational statistics, sampling methods, fast approximation techniques,
- Geoscience applications,
- Design of experiments,
- Decision analysis, value of information,

I like hiking, skiing, tennis, etc.

Plan for course

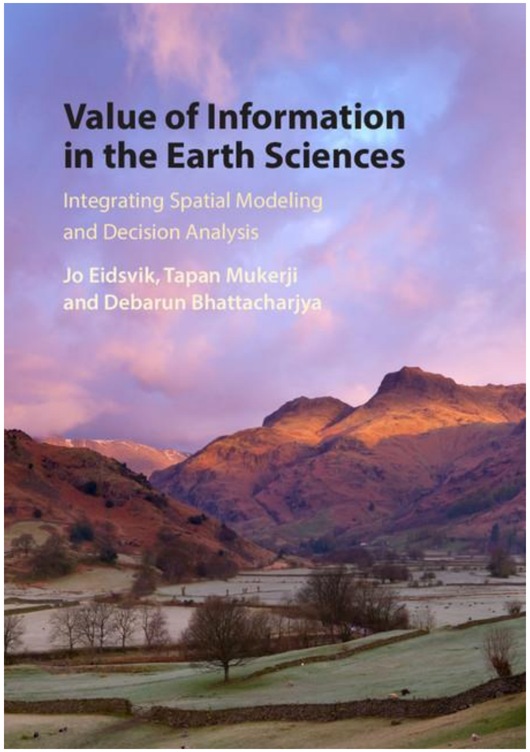
Time	Topic
Monday	Introduction and motivating examples
	Elementary decision analysis and the value of information
Tuesday	Multivariate statistical modeling, dependence, graphs
	Value of information analysis for dependent models
Wednesday	Spatial statistics, spatial design of experiments
	Value of information analysis in spatial decision situations
Thursday	Examples of value of information analysis in Earth sciences
	Computational aspects
Friday	Sequential decisions and sequential information gathering
	Examples from mining and oceanography

Every day: Small exercise half-way, and computer project at the end.

Material:

Relevant background reading :

- Eidsvik, J., Mukerji, T. and Bhattacharjya, D., Value of information in the Earth sciences, Cambridge University Press, 2015.



- Howard R.A. and Abbas, A.E., Foundations of decision analysis, Pearson, 2015.
- Many spatial statisics books:
  - Cressie and Wikle (2011),
  Chiles and Delfiner (2012),
  Banerjee et al. (2014),
  Pyrcz and Deutsch (2014),
  etc.

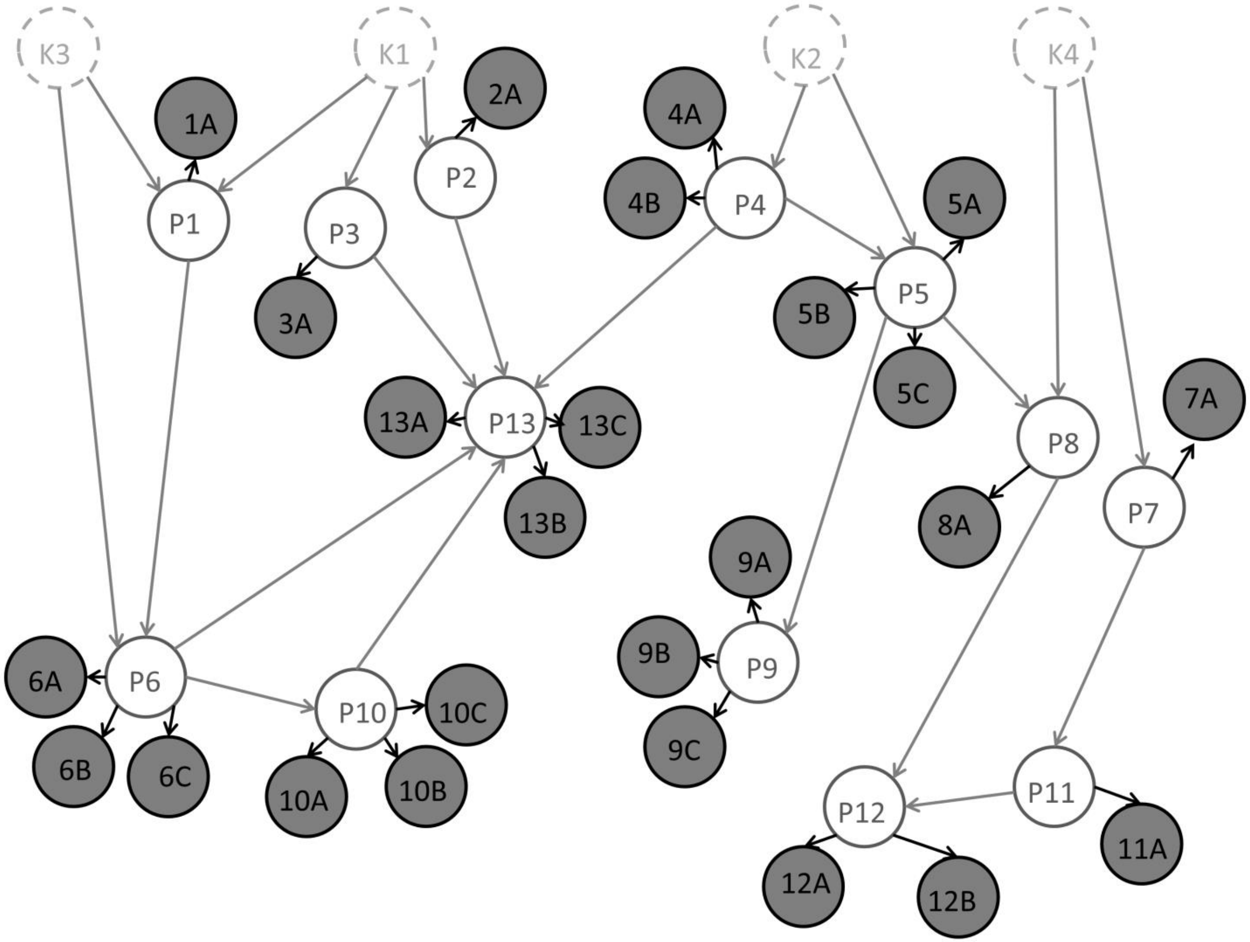
Motivating VOI examples:

Integration of spatial modeling and decision analysis.

Collect data to resolve uncertainties and make informed decisions.

Motivation
(a petroleum exploration example)

Gray nodes are petroleum reservoir segments where the company aims to develop profitable amounts of oil and gas.



Martinelli, G., Eidsvik, J., Hauge, R., and Førland, M.D., 2011, Bayesian networks for prospect analysis in the North Sea, *AAPG Bulletin*, 95, 1423-1442.

Motivation
(a petroleum exploration example)

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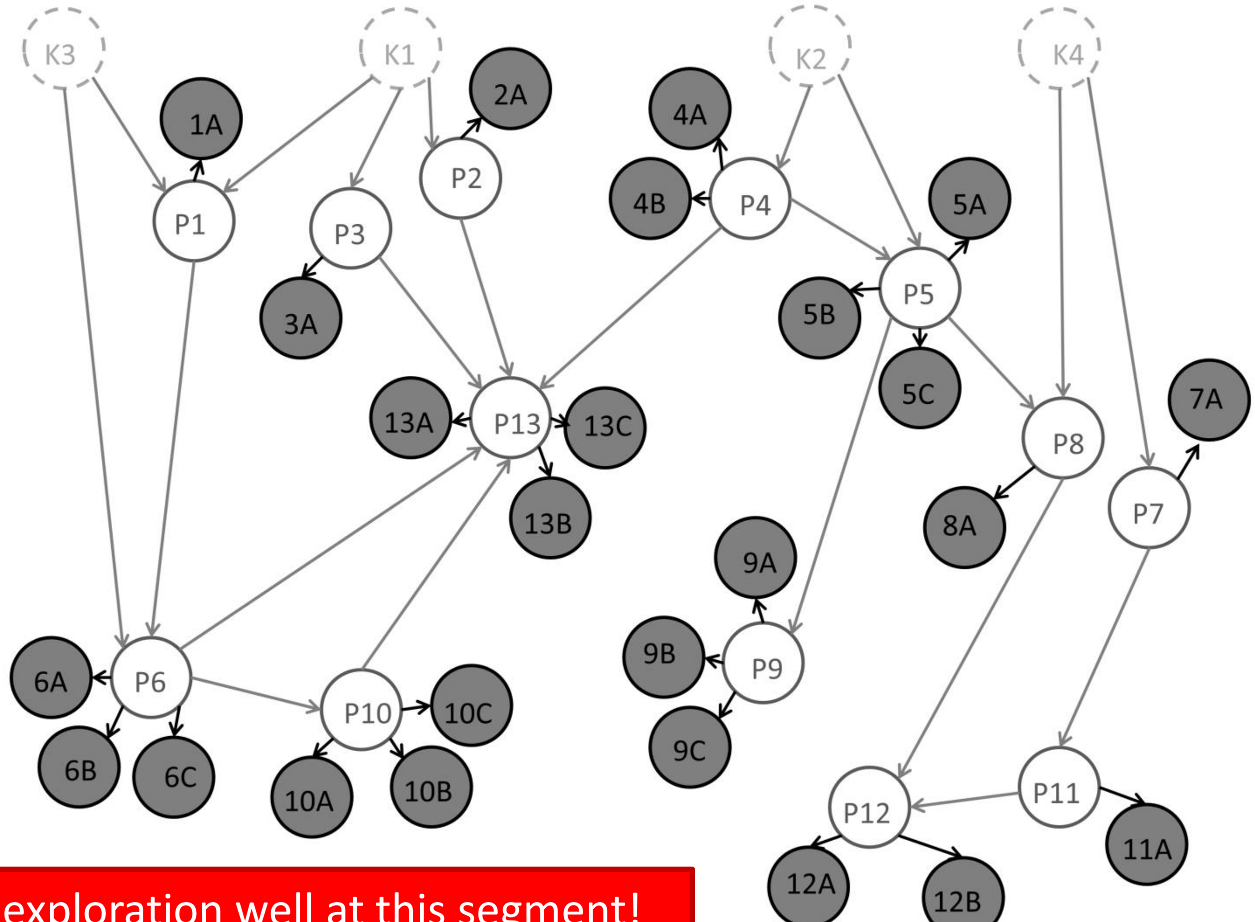
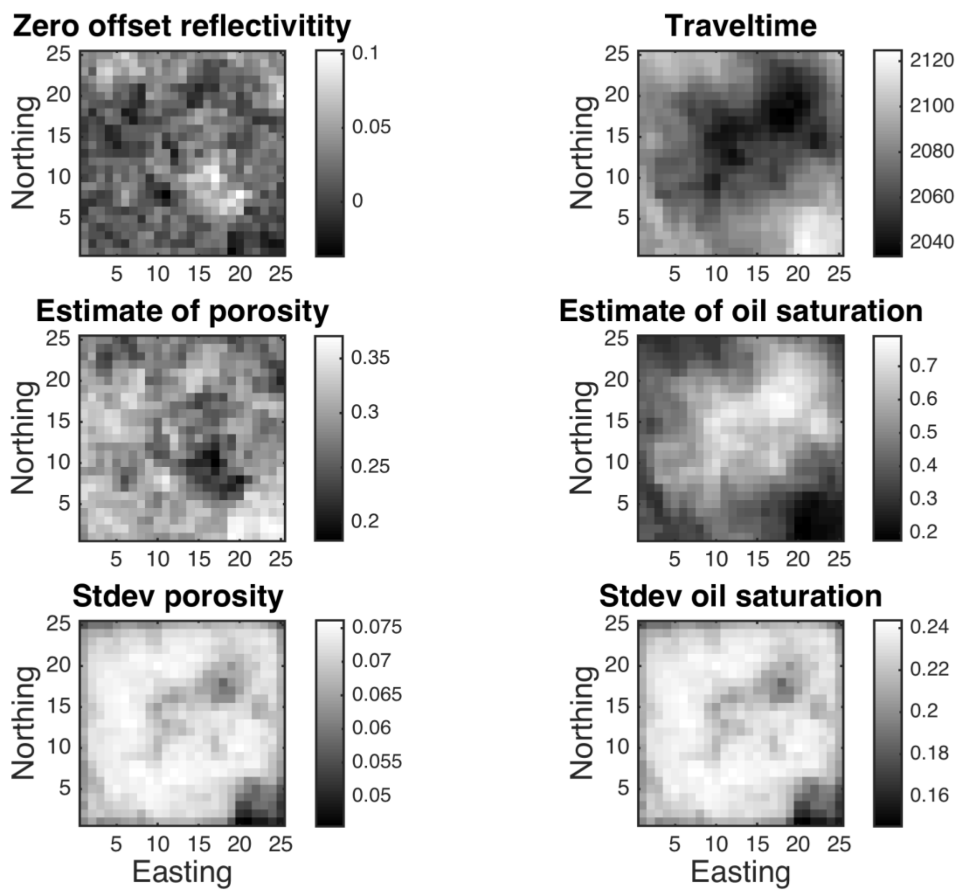


Drill the exploration well at this segment!
The value of information is largest.

Motivation
(a petroleum development example)

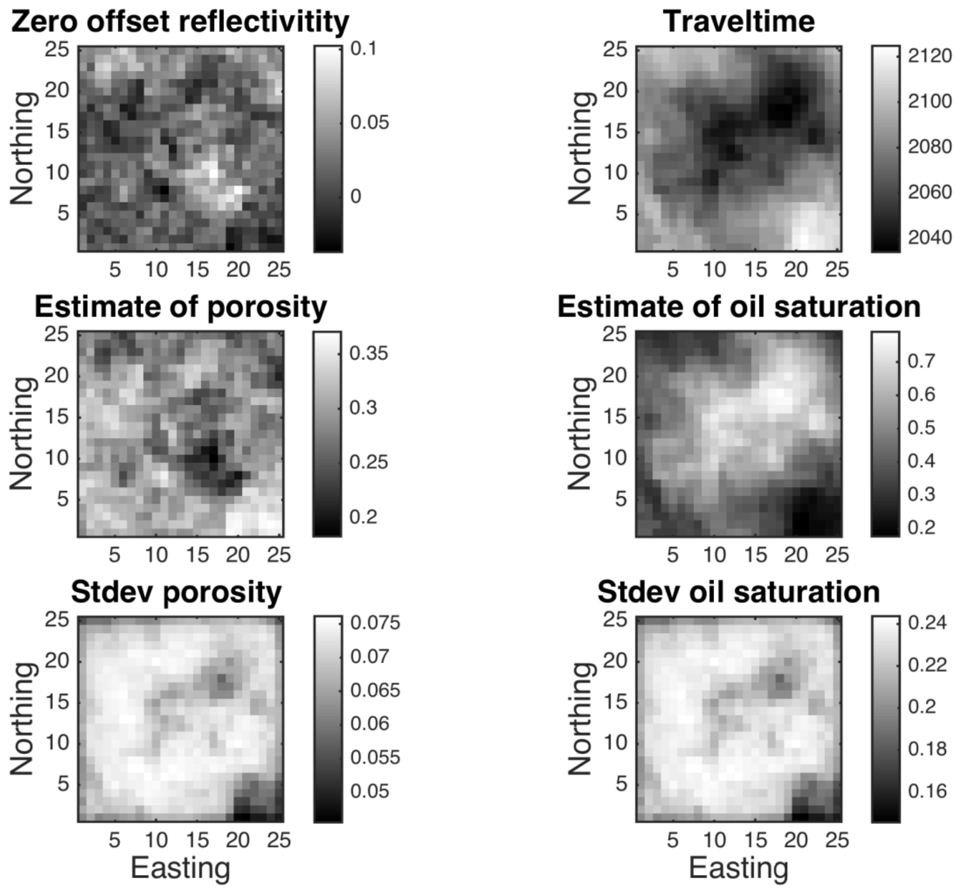
Reservoir predictions from post-stack seismic data!



Eidsvik, J., Bhattacharjya, D. and Mukerji, T., 2008, Value of information of seismic amplitude and CSEM resistivity, *Geophysics*, 73, R59-R69.

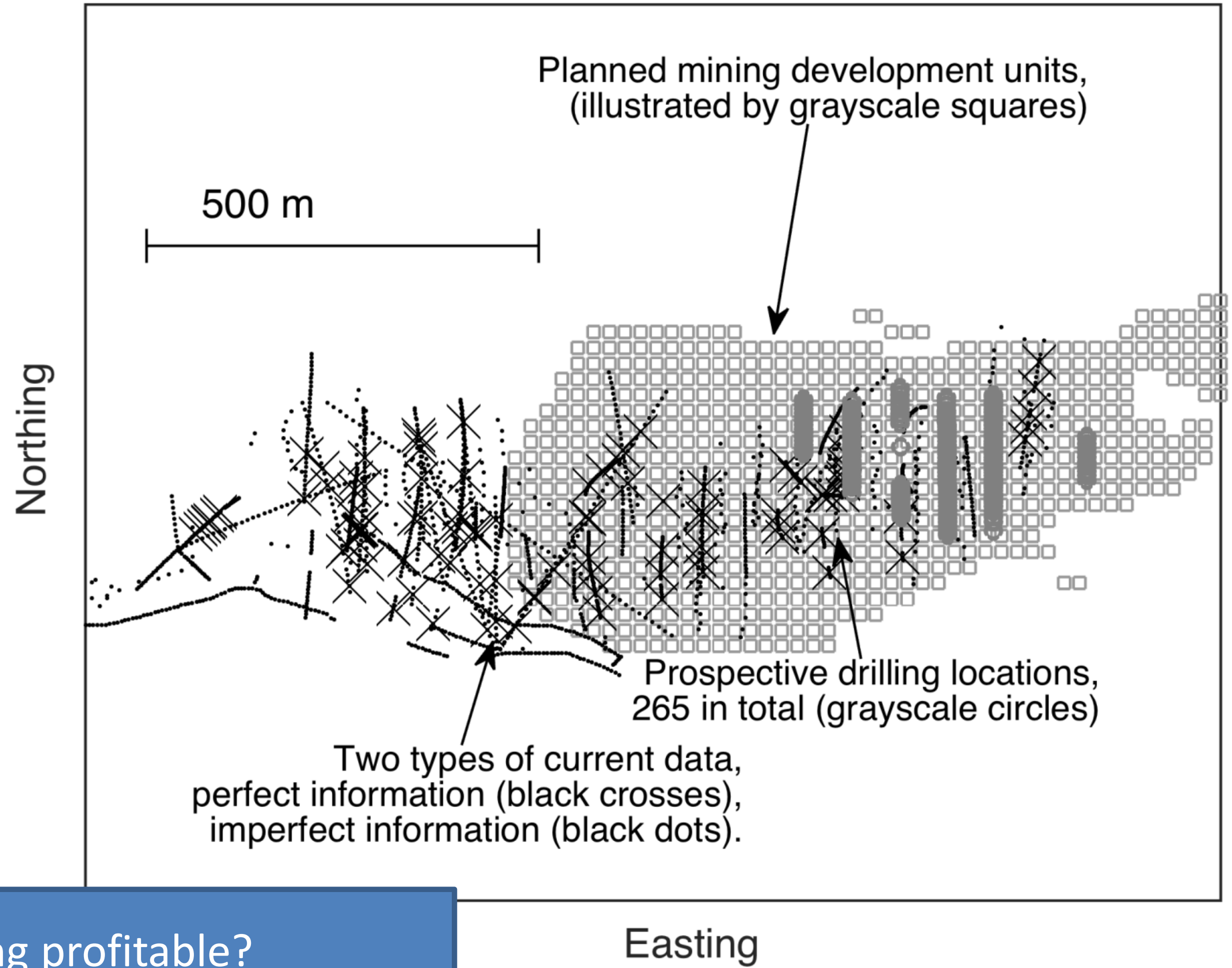
Motivation
(a petroleum development example)

Reservoir predictions from post-stack seismic data!



Process pre-stack seismic data, or electromagnetic data?

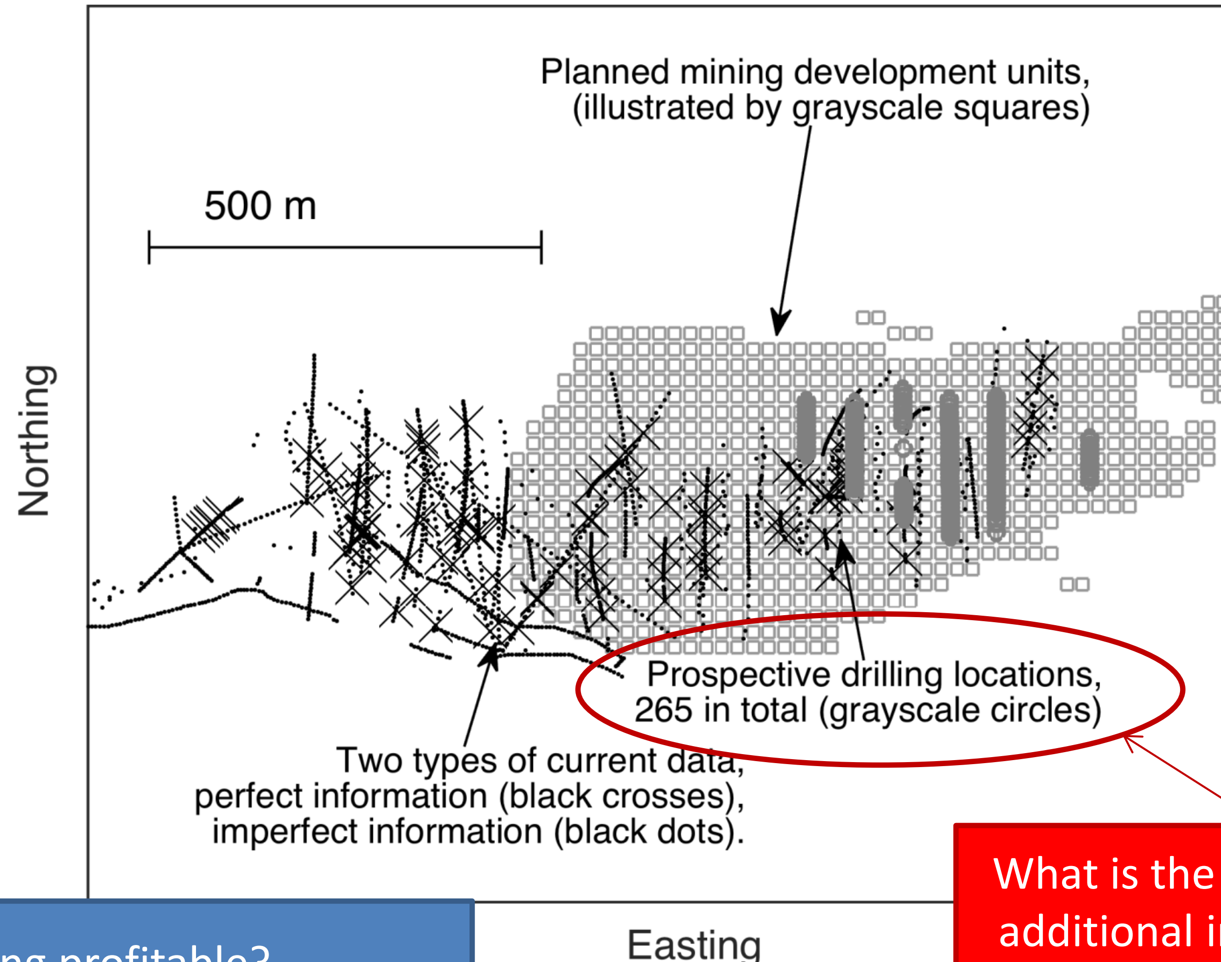
Motivation (an oxide mining example)



Is mining profitable?

Eidsvik, J. and Ellefmo, S.L., 2013, The value of information in mineral exploration within a multi-Gaussian framework, *Mathematical Geosciences*, 45, 777-798.

Motivation (an oxide mining example)

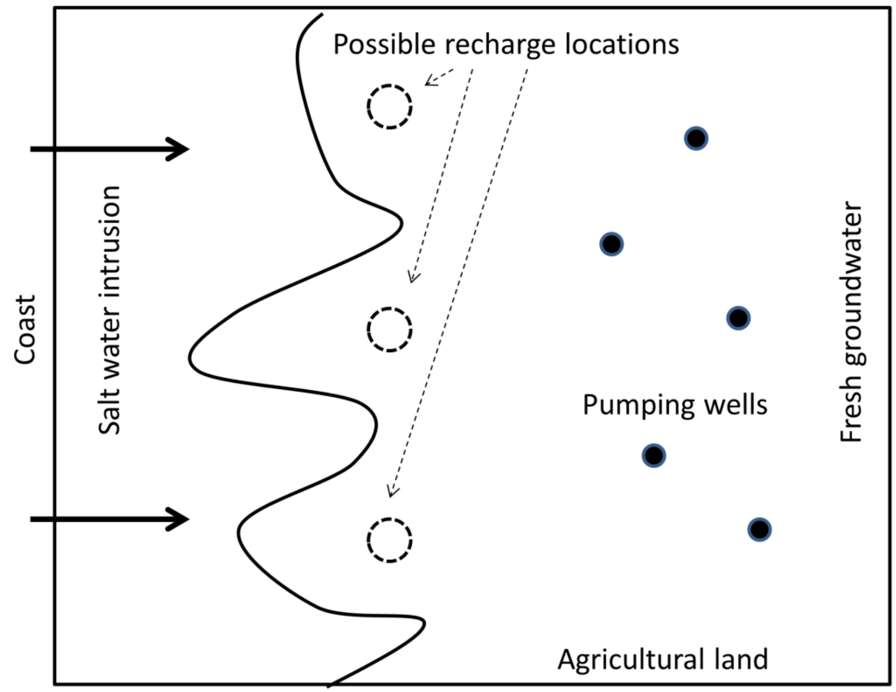


What is the value of this additional information?

Is mining profitable?

Motivation
(a groundwater example)

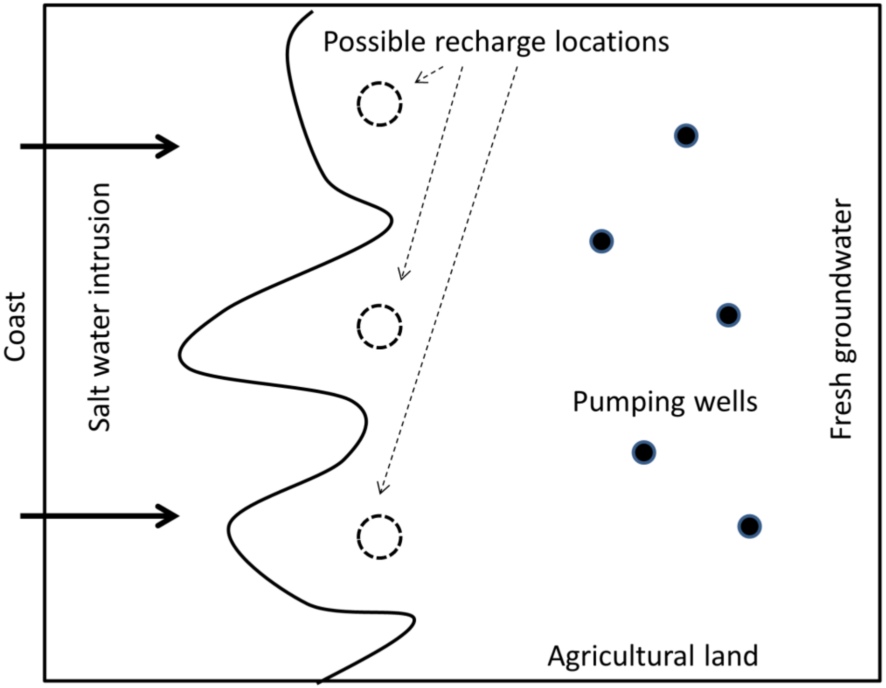
Which recharge location is better to prevent salt water intrusion?



Trainor-Guitton, W.J., Caers, J. and Mukerji, T., 2011, A methodology for establishing a data reliability measure for value of spatial information problems, *Mathematical Geosciences*, 43, 929-949.

Motivation
(a groundwater example)

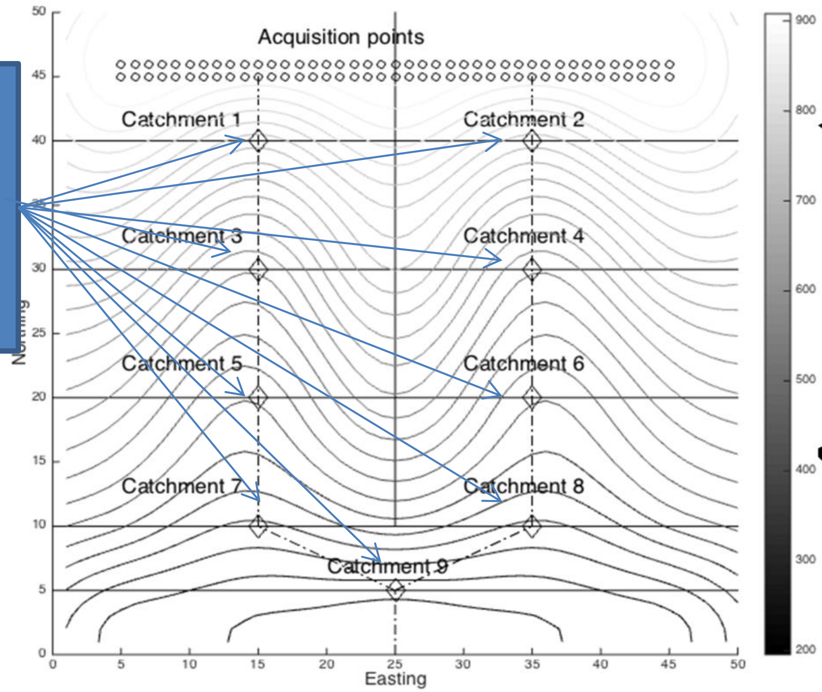
Which recharge location is better to prevent salt water intrusion?



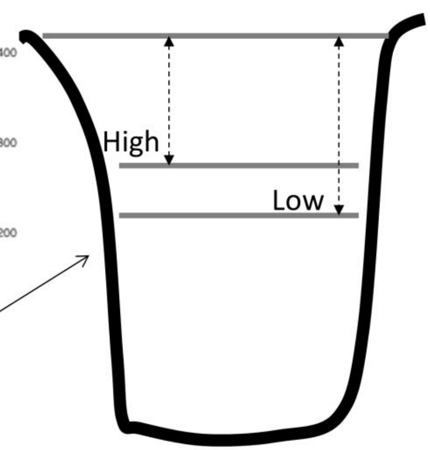
Is it worthwhile to acquire electromagnetic data before making the decision about recharge?

Motivation
(a hydropower example)

Adjusting water levels in 9 hydropower dams!



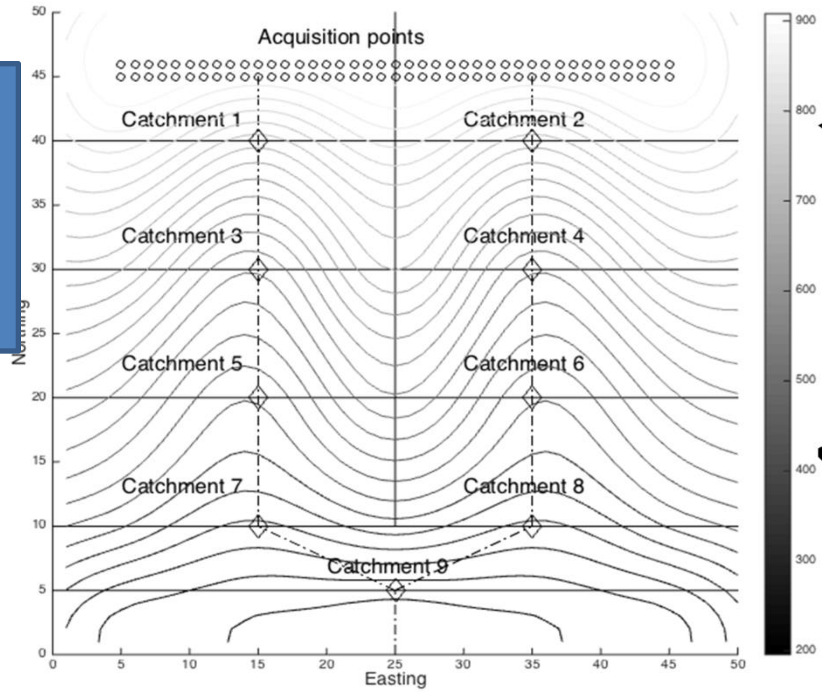
Map view illustration of a river basin. Dams are illustrated by diamonds.



The water level in each dam can be lowered before snow melting.

Motivation
(a hydropower example)

Adjusting water levels in dams!



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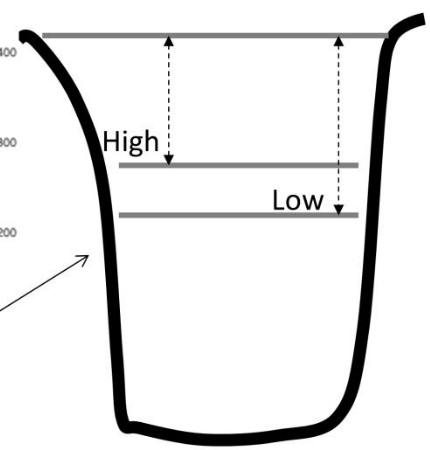


The water level in each dam can be lowered before snow melting.

Acquire snow measurements?

Other applications

- Farming and forestry – how to set up surveys for improved harvesting decisions.
- Biodiversity – where to monitor different biological variables for sustainability.
- Environmental – how monitor where pollutants are, to minimize risk or damage.

- Robotics - where should drone (UAV) or submarine (AUV) go to collect valuable data?
- Industry reliability – how to allocate sensors to 'best' monitor state of system?
- Internet of things – which sensors should be active now?

Which data are valuable?

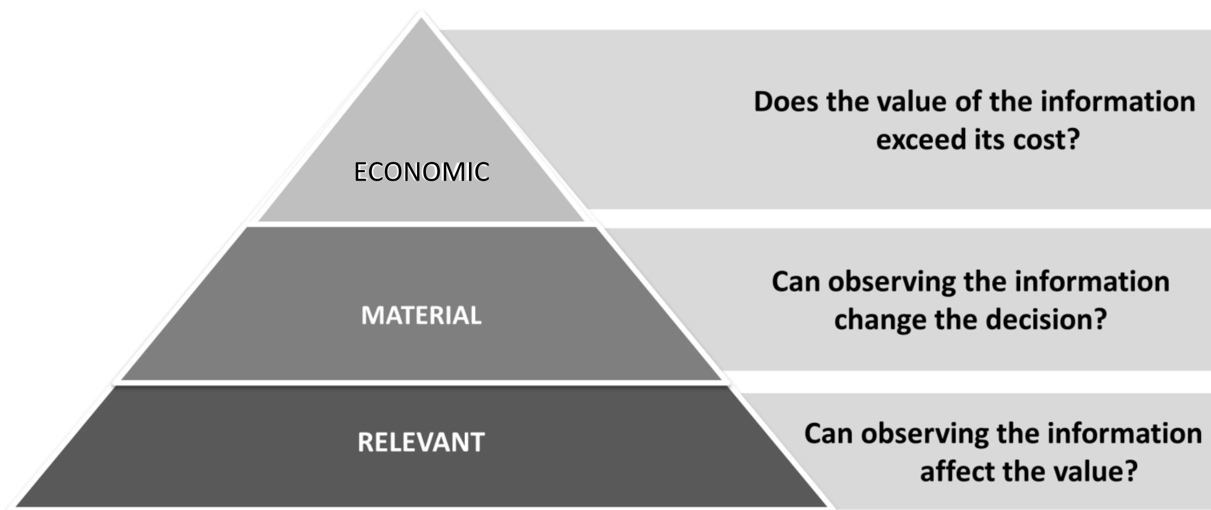
Five Vs of big data:

- Volume
- Variety
- Velocity
- Veracity
- **Value**

We must acquire and process data that has value!
There is often a clear question that one aims to answer, and data should help us.

Value of information (VOI)

In many Earth science applications we consider purchasing more data before making difficult decisions under uncertainty. The value of information (VOI) is useful for quantifying the value of the data, before it is acquired and processed.



This pyramid of conditions - VOI is different from other information criteria (entropy, variance, prediction error, etc.)

Information gathering

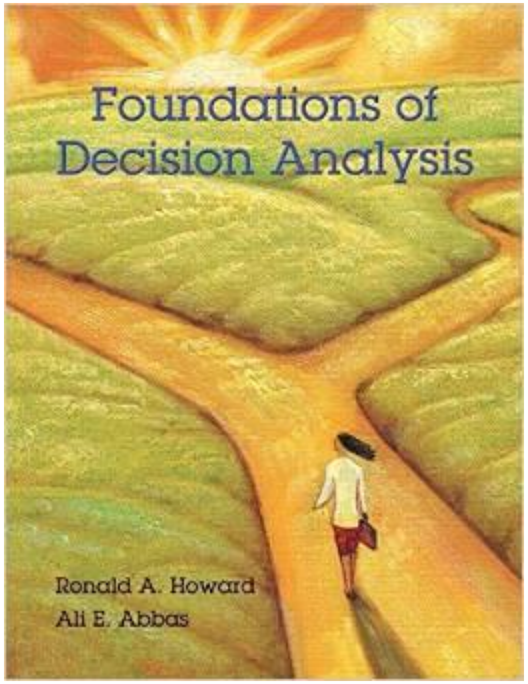
Why do we gather data?

To make better decisions!
To answer some kind of questions!
Reject or strengthen hypotheses!

We will use a **decision theoretic** perspective, but the methods are easily adapted to other criteria or value functions (Wednesday).

Decision analysis (DA)

Decision analysis attempts to guide a decision maker to clarity of action in dealing with a situation where one or more decisions are to be made, typically in the face of uncertainty.



Howard, R.A. and Abbas, A., 2015, *Foundations of Decision Analysis*, Prentice Hall.

Framing a decision situation

Rules of actional thought. (Howard and Abbas, 2015)

- Frame your decision situation to address the decision makers true concerns.
- Base decisions on maximum expected utility.

‘…systematic and repeated violations of these principles will result in inferior long-term consequences of actions and a diminishes quality of life…’

(Edwards et al., 2007, Advances in decision analysis: From foundations to applications, Cambridge University Press.)

Pirate example
(For motivating decision analysis and VOI)

Pirate example

- **Pirate example**: A pirate must decide whether to dig for a treasure, or not. The treasure is absent or present (uncertainty).

?

Pirate makes decision based on preferences and maximum **utility** or **value**!

- Digging cost.
- Revenues if he finds the treasure .

Pirate example

$a \in \{0,1\}$

- **Pirate example**: A pirate must decide whether to dig for a treasure, or not. The treasure is absent or present (uncertainty).

?

$x \in \{0,1\}$

Pirate makes decision based on preferences and maximum **utility** or **value**!

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$$\max_{a \in \{0,1\}} \left\{ E\left(v\left(x,a\right)\right) \right\}$$

Mathematics of decision situation:

- **Alternatives**

$$a \in \{0,1\} = A$$

- **Uncertainties (probability distribution)**

$$x \in \{0,1\} = \Omega \qquad p(x=1) = 0.01$$

- **Values**

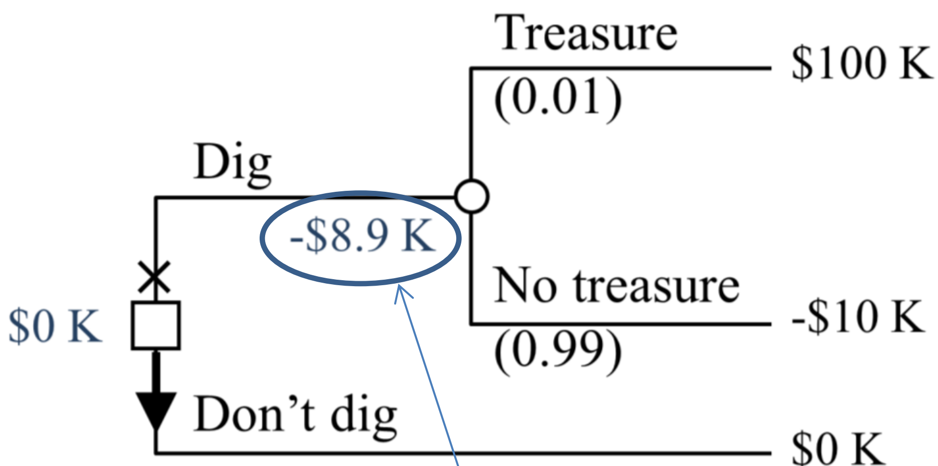
$$v = v(x,a)$$

$$v(x=0, a=1) = -10000 \qquad v(x=1, a=1) = 100000 \qquad v(x, a=0) = 0$$

- **Maximize expected value**

$$a^* = \arg\max_{a \in A} \{E(v(x,a))\}$$

Pirate's decision situation

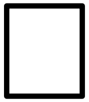


Risk neutral!

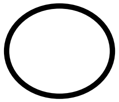
$$E\left(u\left(v_{dig}\right)\right) = E\left(v_{dig}\right) = 0.01(100000) + 0.99(-10000) = -8900$$

Decision trees

A way of structuring and illustrating a decision situation.



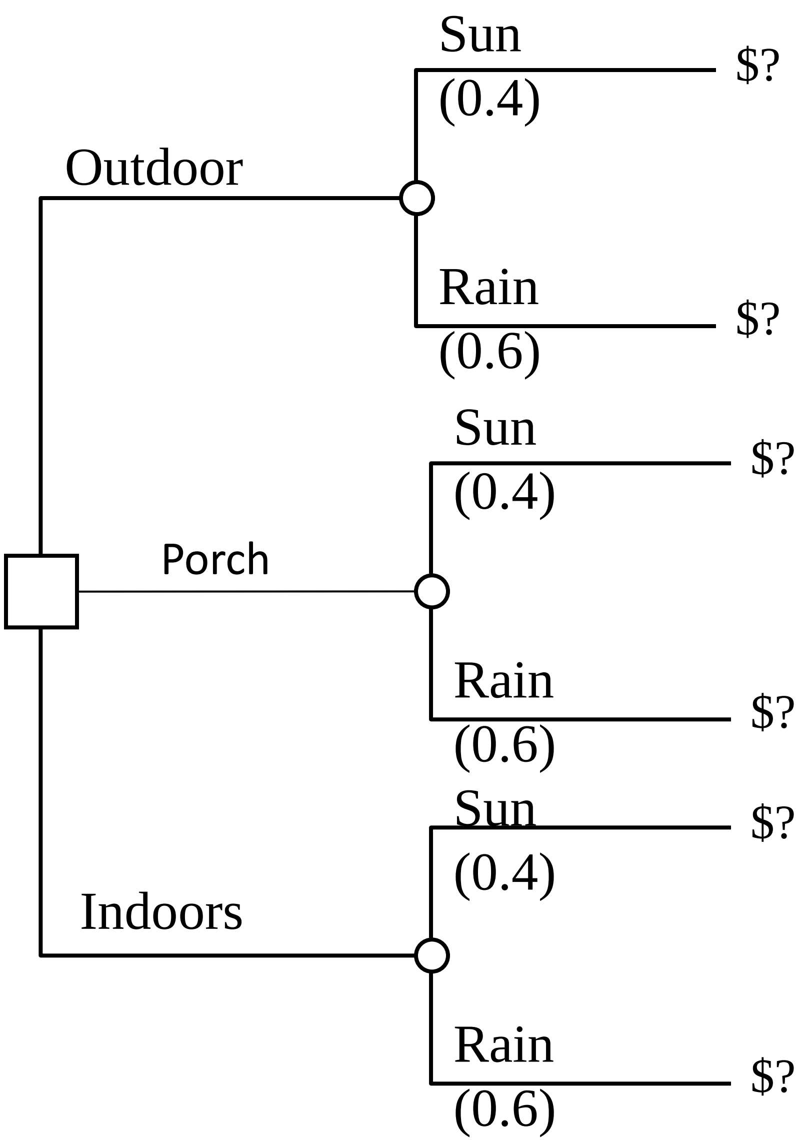
- Squares represent decisions
- Circles represent uncertainties



- Probabilities and values are shown by numbers.
- Arrows indicate the optimal decision.

Kim's party problem

Howard, R.A. and Abbas, A., 2015, *Foundations of Decision Analysis*, Prentice Hall.



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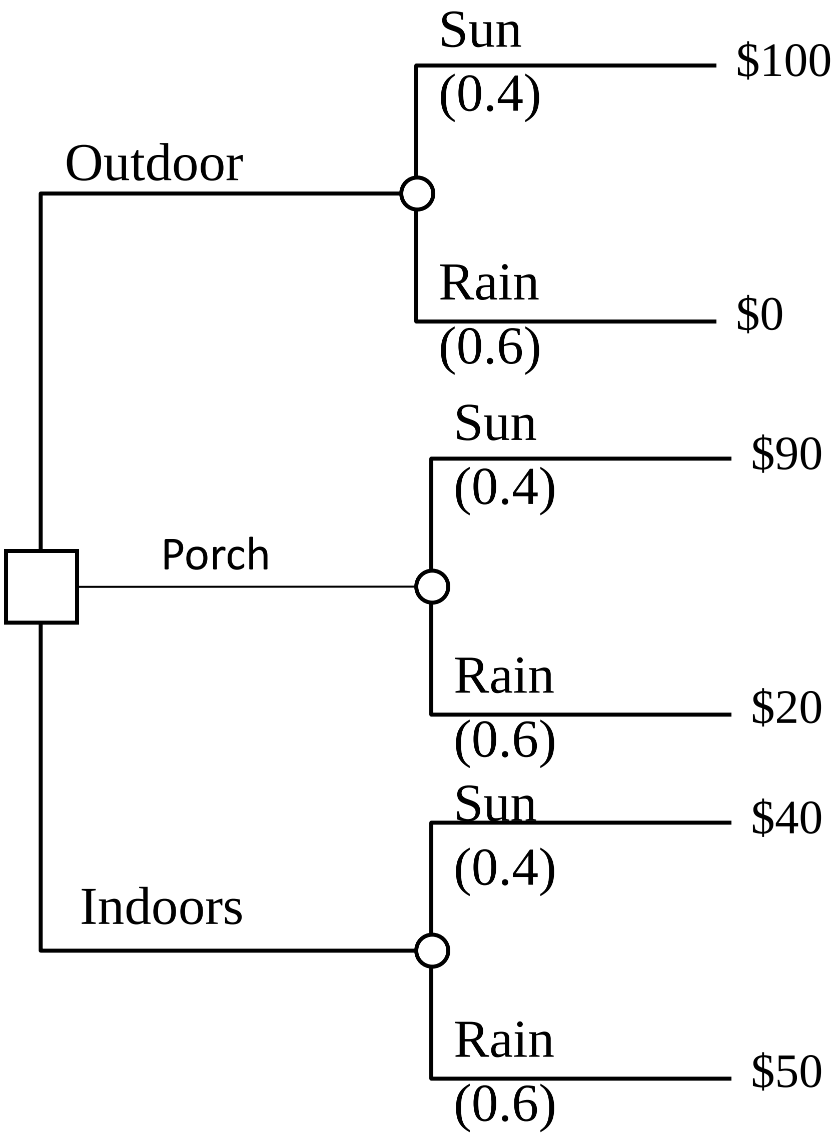

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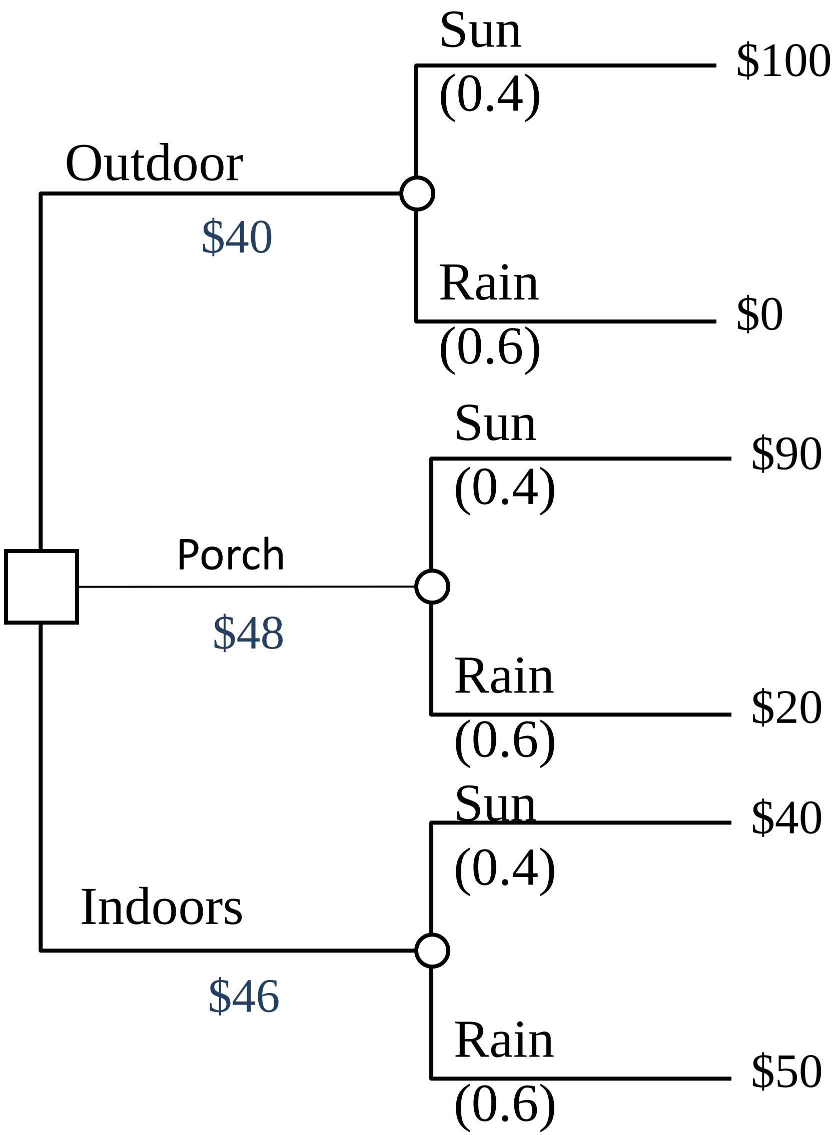

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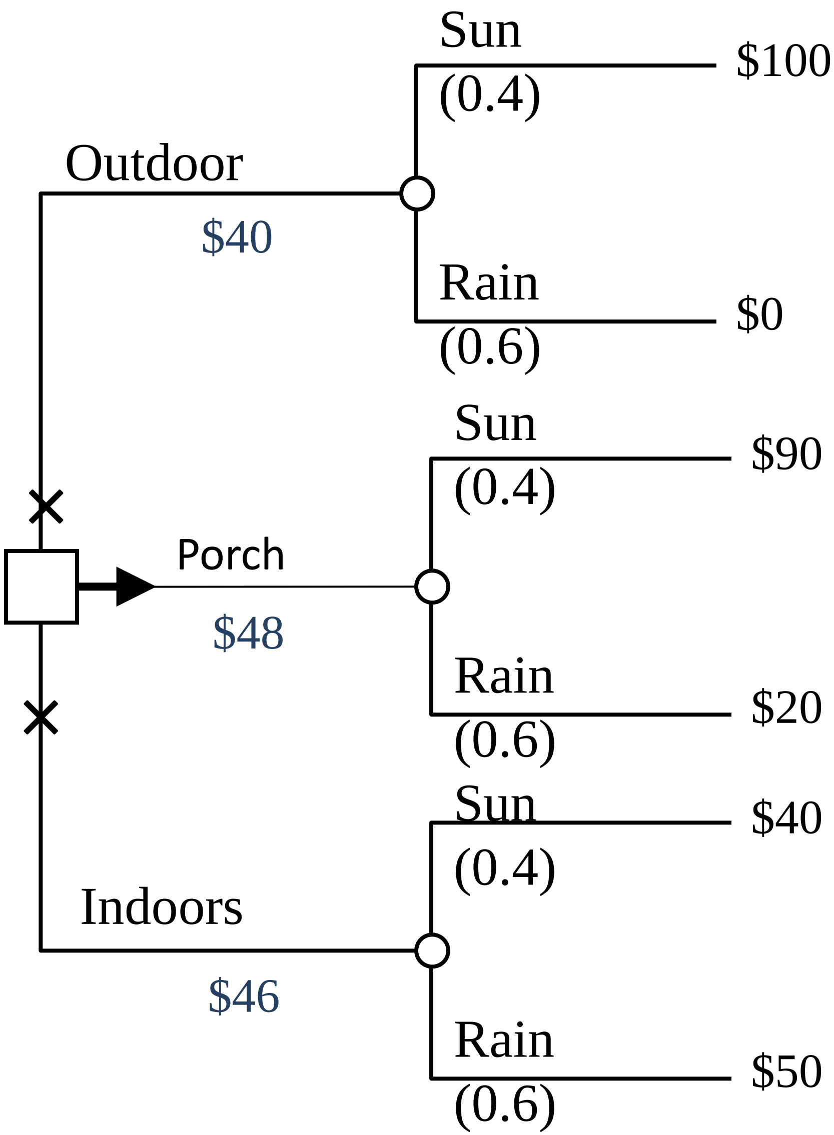

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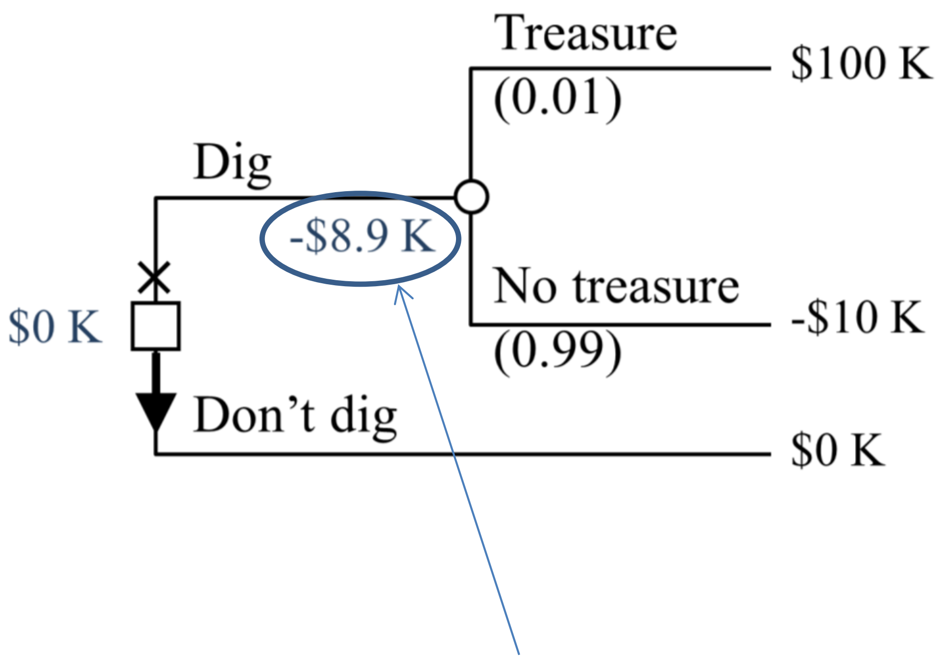


$$E\left(u\left(v_{dig}\right)\right) = E\left(v_{dig}\right) = 0.01(100000) + 0.99(-10000) = -8900$$

Pirate example

- **Pirate example**: A pirate must decide whether to dig for a treasure, or not. The treasure is absent or present (uncertainty).

- Pirate can collect **data** before making the decision, if the experiment is worth its price!

- Perfect information.
Clairvoyant!

- Imperfect information.
Detector!

Value of information (VOI)

- VOI analysis is used to compare the additional value of making informed decisions with the price of the information.
- If the VOI exceeds the price, the decision maker should purchase the data.

VOI=Posterior value – Prior value

VOI – Pirate considers clairvoyant

$$PV = 0 = \$0K$$

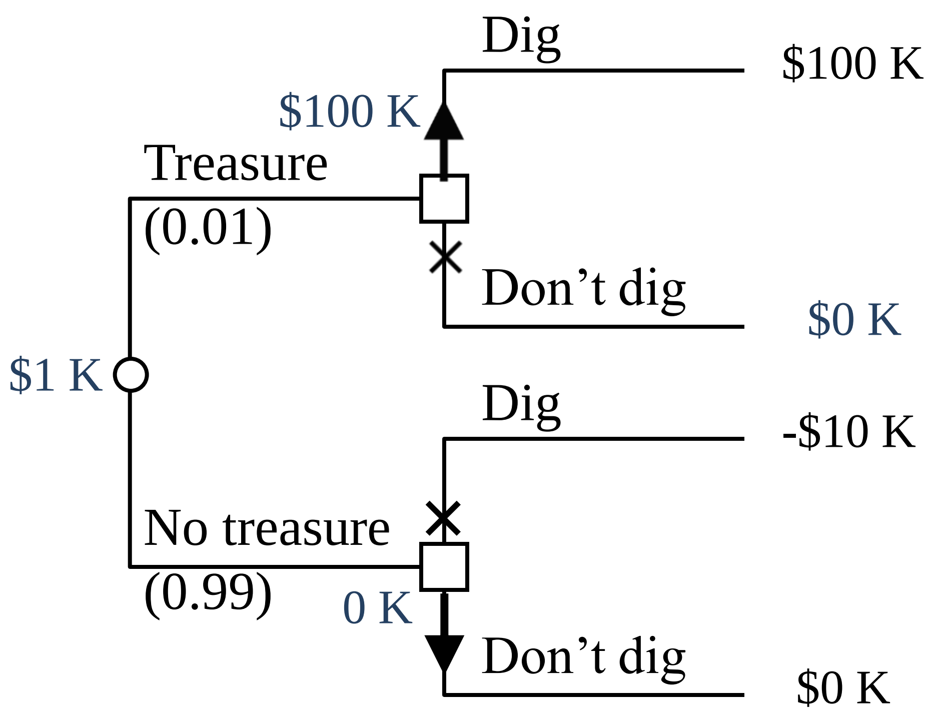
$$PoV(x) = \sum_x \max_{a \in A} \{v(x,a)\} p(x)$$

$$= \left(0.01 \cdot \max\{0,100\}\right) + \left(0.99 \cdot \max\{0,-10\}\right) = \$1K$$

$$VoI(x) = PoV(x) - PV = 1 - 0 = \$1K$$

Conclusion: Consult clairvoyant if (s)he charges less than $1000.

PoV – decision tree, perfect information



Pirate example - detector

- **Pirate example**: A pirate must decide whether to dig for a treasure, or not. The treasure is absent or present (uncertainty).
- Pirate can collect **data** before making the decision, if the experiment is worth its price!

Pirate makes decision based on preferences and maximum expected **value**!

- Digging cost.
- Revenues if he finds the treasure .

Pirate example - detector

$a \in \{0,1\}$

- **Pirate example**: A pirate must decide whether to dig for a treasure, or not. The treasure is absent or present (uncertainty).

- Pirate can collect **data** with a detector before making the decision, if this experiment is worth its price!

$x \in \{0,1\}$

$y \in \{0,1\}$

Pirate makes decision based on preferences and maximum expected **value**!
- Digging cost.
- Revenues if he finds the treasure .

$$\max_{a\in\{0,1\}} \{E(v(x,a) \mid y)\}$$

Detector experiment

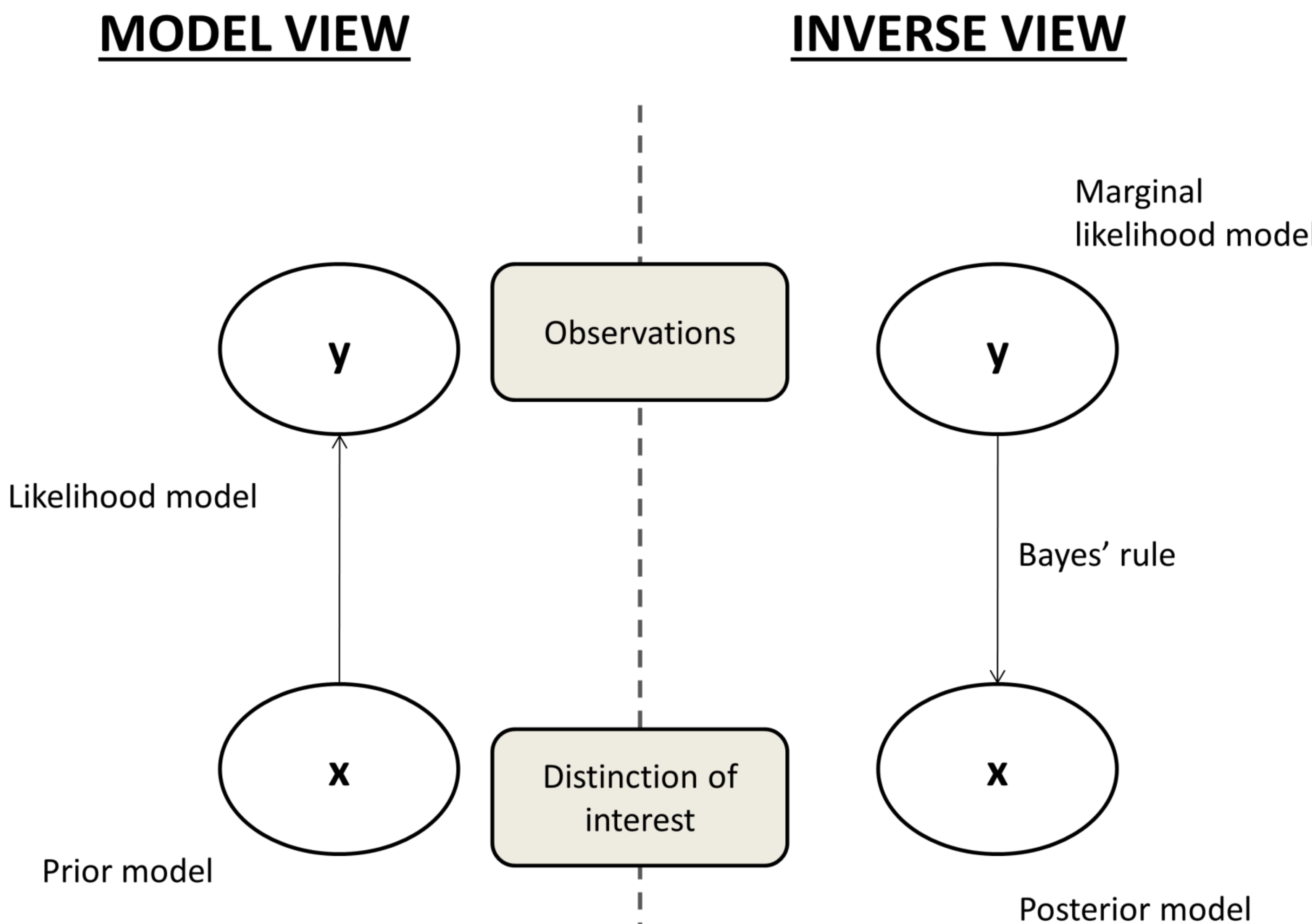
Accuracy of test:

$$p(y=0\,|\,x=0) = p(y=1\,|\,x=1) = 0.95$$

Should the pirate pay to do a detector experiment?

Does the VOI of this experiment exceed the price of the test?

Bayes rule - Detector experiment



Bayes rule - Detector experiment

Likelihood:

$$p(y=0 \mid x=0) = p(y=1 \mid x=1) = 0.95$$

Marginal likelihood:

$$p\left(y=1\right) = p\left(y=1 \mid x=0\right) p\left(x=0\right) + p\left(y=1 \mid x=1\right) p\left(x=1\right)$$
$$= 0.05 \cdot 0.99 + 0.95 \cdot 0.01 = 0.06$$

Posterior:

$$p(x=1 \mid y=1) = \frac{p(y=1 \mid x=1)\, p(x=1)}{p(y=1)} = \frac{0.95 \cdot 0.01}{0.06} \approx 0.16 = 16/100.$$

$$p(x=1 \mid y=0) = \frac{p(y=0 \mid x=1)\, p(x=1)}{p(y=0)} = \frac{0.05 \cdot 0.01}{0.94} \approx 0.0005 = 5/10000.$$

VOI – Pirate considers detector test

$$PoV(y) = \sum_y \max_{a \in A} \left\{ E\left(v(x,a) \mid y \right) \right\} p(y)$$

$$= \left(0.06 \cdot \max\left\{ 0, (100 \cdot 0.16) + (-10 \cdot 0.84) \right\} \right)$$

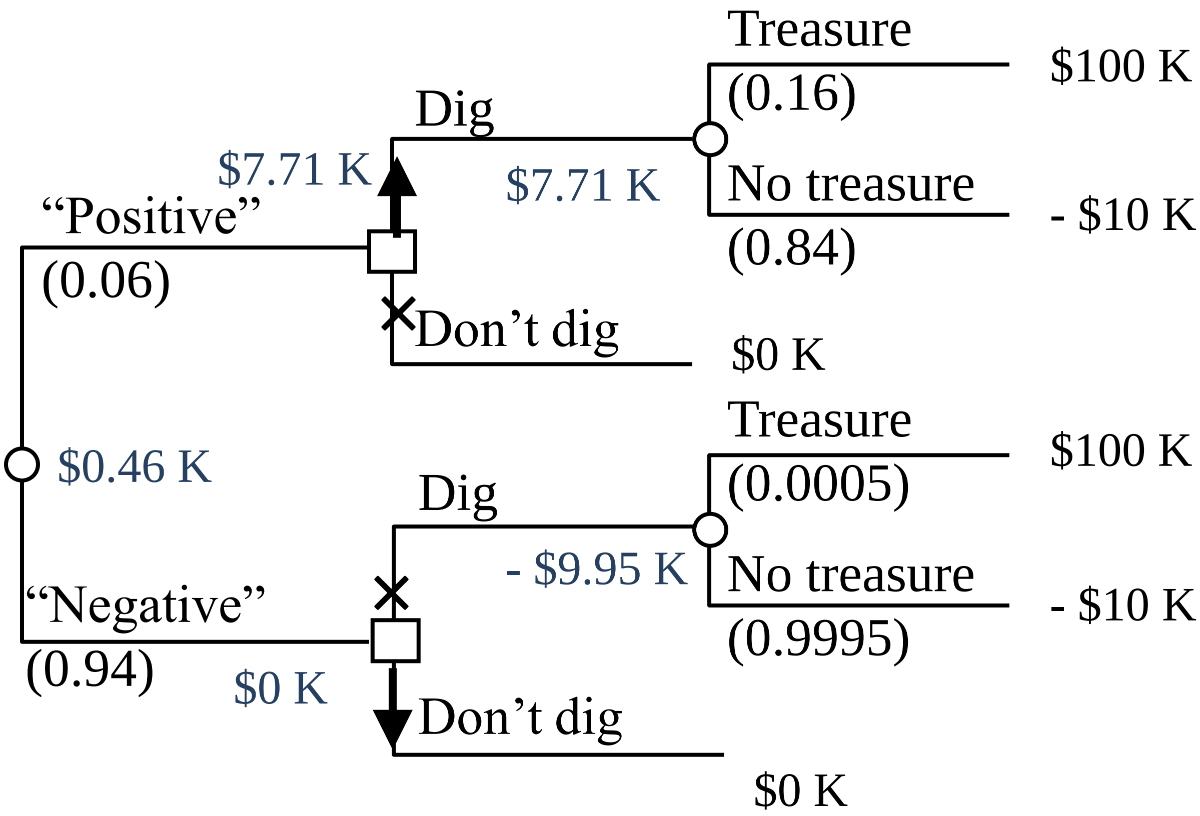
$$+ \left(0.94 \cdot \max\left\{ 0, (100 \cdot 0.0005) + (-10 \cdot 0.9995) \right\} \right)$$

$$= \left(0.06 \cdot \max\left\{ 0, 7.71 \right\} \right) + \left(0.94 \cdot \max\left\{ 0, -9.95 \right\} \right) = \$0.46K.$$

$$VoI(y) = PoV(y) - PV = 0.46 - 0 = \$0.46K$$

Conclusion: Purchase detector testing if its price is less than $460.

PoV - imperfect information

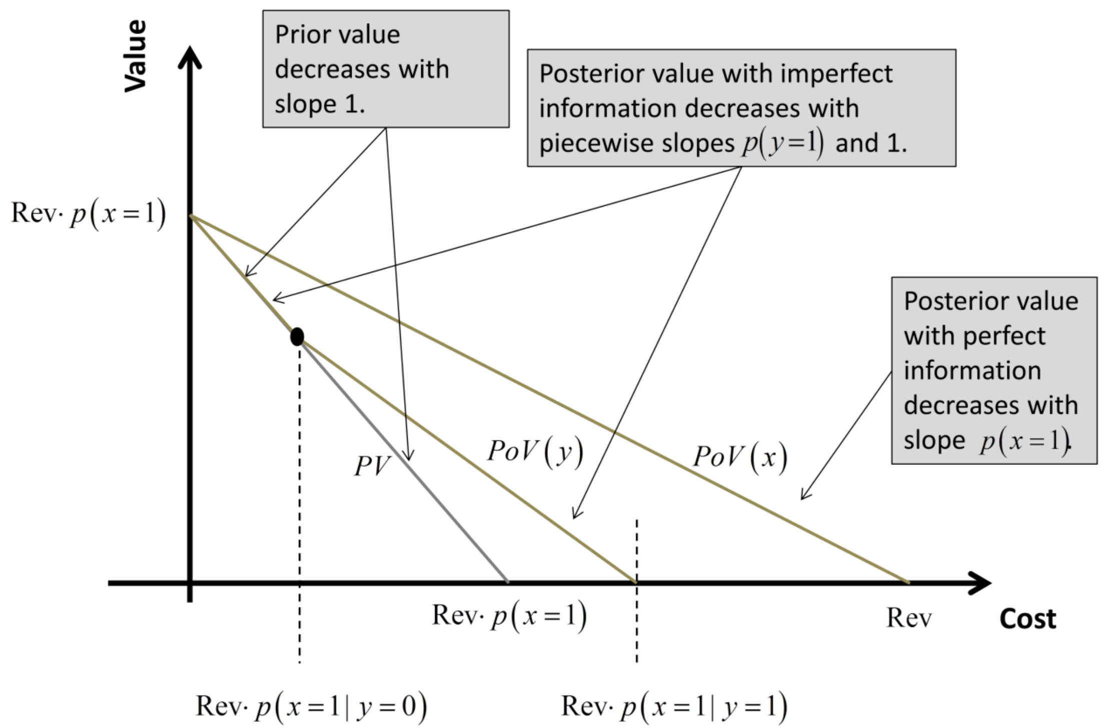


PV and PoV as a function of Digging Cost

$$PV = \max\left\{0, \text{Rev}\cdot p\left(x=1\right) - \text{Cost}\right\}$$

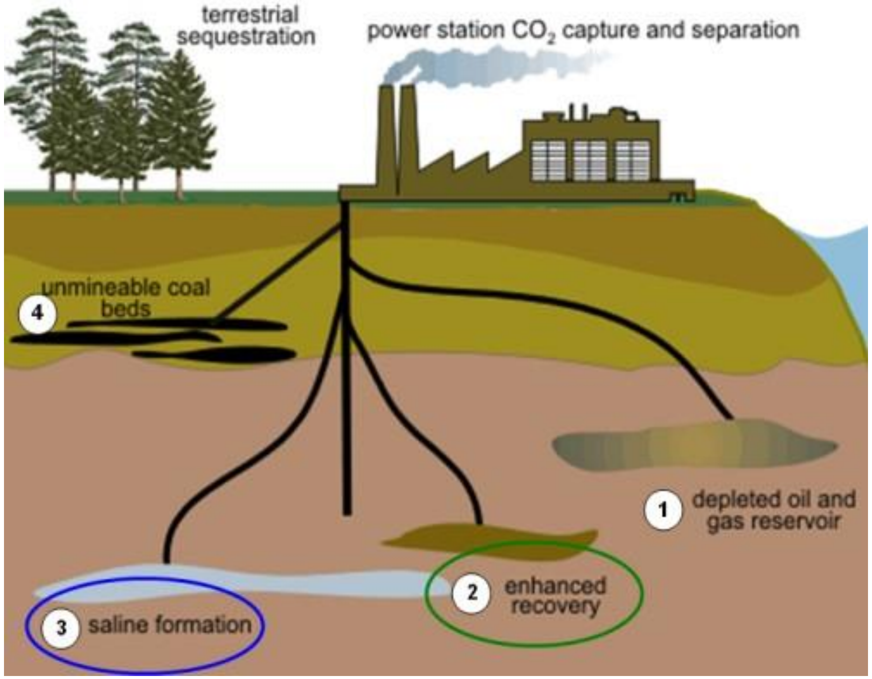
$$PoV\left(x\right) = \max\left\{0, \text{Rev} - \text{Cost}\right\} p\left(x=1\right)$$

$$PoV\left(y\right) = \sum_{y} \max\left\{0, \text{Rev}\cdot p\left(x=1 \mid y\right) - \text{Cost}\right\} p\left(y\right)$$



Exercise: CO_2 sequestration

CO_2 is sequestered to reduce carbon emission in the athmosphere and defer global warming.
Geological sequestration involves pumping CO_2 in subsurface layers, where it will remain, unless it leaks to the surface.



VOI for CO2 sequestration

The decision maker can proceed with CO2 injection or suspend sequestration. The latter incurs a tax of 80 monetary units. The former only has a cost of injection equal to 30 monetary units, but the injected CO2 may leak (x=1). If leakage occurs, there will be a fine of 60 monetary units (i.e. a cost of 90 in total). Decision maker is risk neutral.

$$p(x=1)=0.3 \qquad p(x=0)=0.7$$

Data: Geophysical experiment, with binary outcome, indicating whether the formation is leaking or not.

$$p(y=0|x=0)=0.95 \qquad p(y=1|x=1)=0.9$$

Exercise:

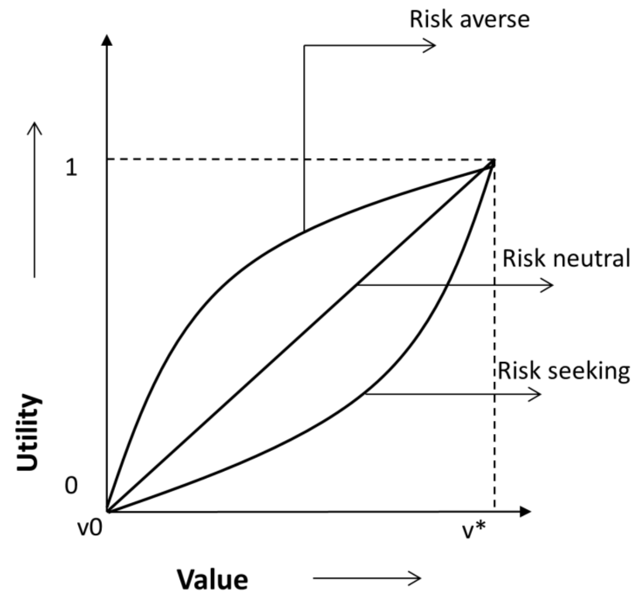
1. **Draw the decision tree without information.**
2. **Draw the decision tree with perfect information (clairvoyance).**
3. **Compute the VOI of perfect information.**
4. **Draw the decision tree with the geophysical experiment.**
5. **Compute conditional probabilities, expected values and the VOI of geophysical data.**

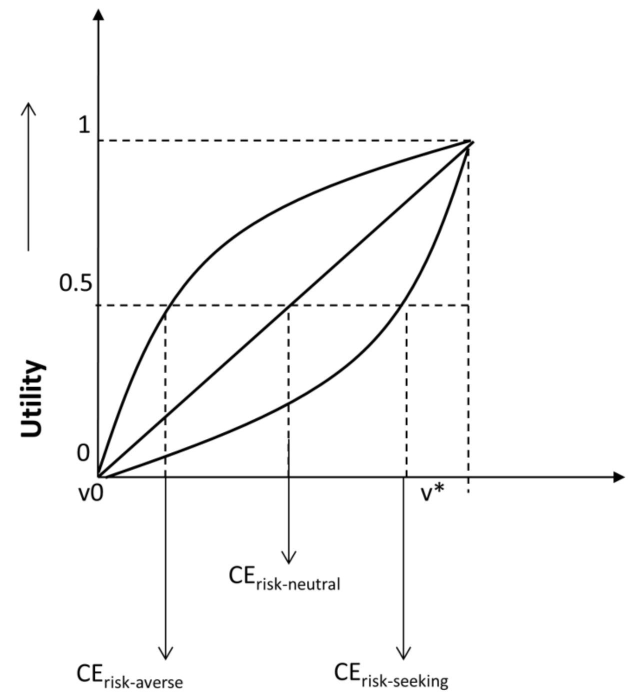
Value of information (VOI)
- More general formulation

- VOI analysis is used to compare the additional value of making informed decisions with the price of the information.
- If the VOI exceeds the price, the decision maker should purchase the data.

VOI=Posterior value – Prior value

Risk and utility functions



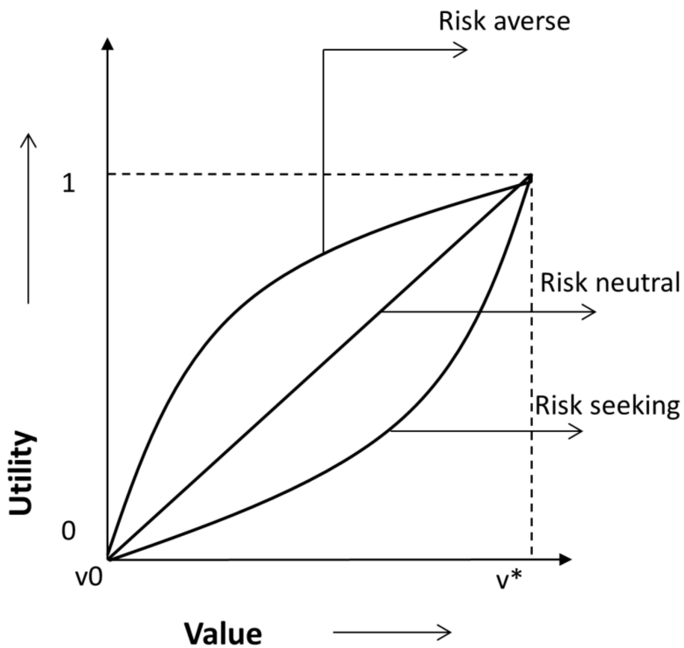


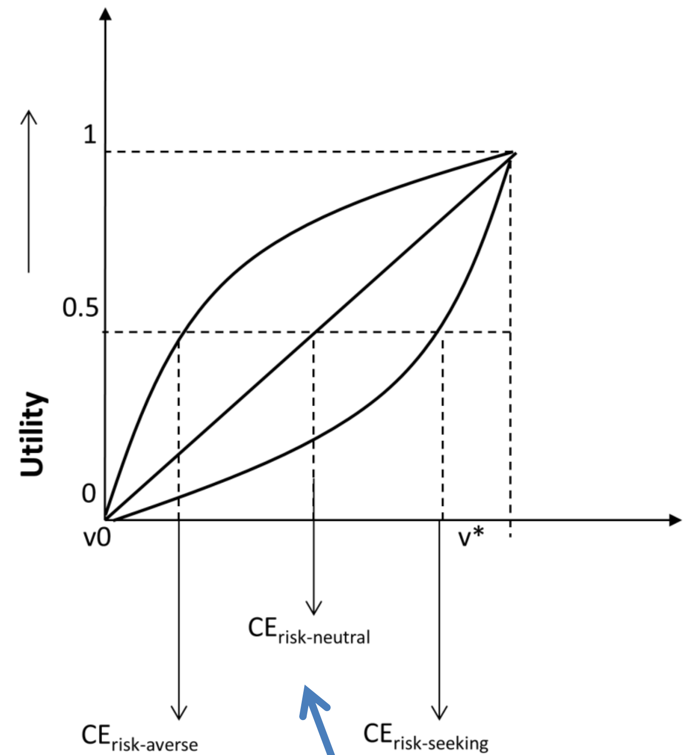
Exponential and linear utility have constant risk aversion coefficient:

$$\gamma = -{u''(v)}/{u'(v)}$$

Certain equivalents (CE)

Utilities are mathematical. The certain equivalent is a measure of how much a situation is worth to the decision maker. (It is measured in value).





$$CE = u^{-1}\left(\max\left\{E\left(u\left(v_{dig}\right)\right), E\left(u\left(v_{don't\ dig}\right)\right)\right\}\right)$$

What is the value of indifference? How much would the owner of a lottery be willing to sell it for?

VOI - Clairvoyance

Price *P* of experiment makes the equality.

$$\sum_x \max_{a\in A}\left\{v\left(x,a\right)-P\right\}p\left(x\right)=\max_{a\in A}\left\{E\left(v\left(x,a\right)\right)\right\}$$

$$\rightarrow P=VOI=\sum_x \max_{a\in A}\left\{v\left(x,a\right)\right\}p\left(x\right)-\max_{a\in A}\left\{E\left(v\left(x,a\right)\right)\right\}$$

VOI=Posterior value – Prior value

Assuming risk neutral decision maker!

Value of information- Imperfect

Price of indifference.

$$\sum_y \max_{a\in A}\left\{E\left(v\left(x,a\right)-P\ y\right)\right\}p\left(y\right)=\max_{a\in A}\left\{E\left(v\left(x,a\right)\right)\right\}$$

$$\sum_y \max_{a\in A}\left\{E\left(v\left(x,a\right)-P\mid y\right)\right\}p\left(y\right)=\max_{a\in A}\left\{E\left(v\left(x,a\right)\right)\right\}$$

$$\rightarrow P=VOI=\sum_y \max_{a\in A}\left\{E\left(v\left(x,a\right)\mid y\right)\right\}p\left(y\right)-\max_{a\in A}\left\{E\left(v\left(x,a\right)\right)\right\}$$

VOI=Posterior value – Prior value

Assuming risk neutral decision maker!

Properties of VOI

a) VOI is always positive

- Data allow better, informed decisions.

$$\max\left\{0, \sum_i v_i\right\} \leq \sum_i \max\{0, v_i\}$$

b) If value is in monetary units ,VOI is in monetary units.

c) Data should be purchased if VOI > Price of experiment P.

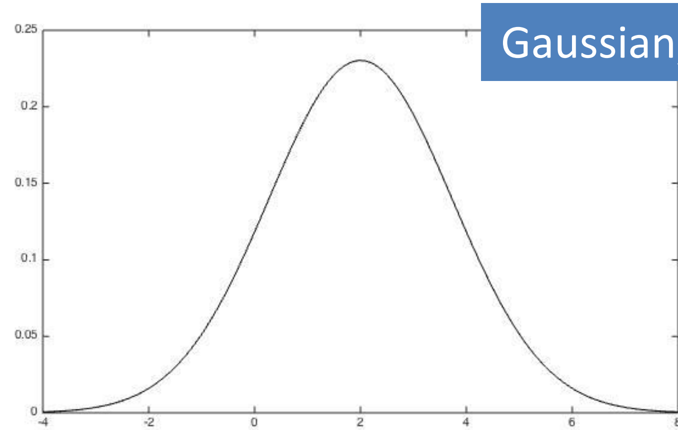
d) VOI of clairvoyance is an upper bound for any imperfect information gathering scheme.

e) When we compare different experiments, we purchase the one with largest VOI compared with the price:

$$\arg\max\{VOI_1 - P_1, VOI_2 - P_2\}$$

Gaussian model for profits

$$p(x) = \frac{1}{\sqrt{2\pi r^2}} \exp\left(-\frac{(x-m)^2}{2r^2} \right)$$



Gaussian, m=2, r=3

Uncertain profits of a project is Gaussian distributed.

VOI for Gaussian

Uncertain project profit is Gaussian distributed.
Invest or not?
The decision maker asks a clairvoyant for perfect information, if the VOI is larger than her price.

$$VOI(x) = \text{PosteriorValue}(x) - \text{PriorValue}$$

$$PV = \max\{0, E(x)\}, \qquad E(x) = m$$

$$PoV(x) = E(\max\{0, x\}) = \int \max\{0, x\}\, p(x)\, dx$$

VOI for Gaussian

Result:

$$E\left(\max\{0,x\}\right)=\int \max\{0,x\}\,p(x)\,dx=\int_{0}^{\infty} x p(x)\,dx=\int_{-m/r}^{\infty}(m+rz)\,\phi(z)\,dz$$

$$=m\int_{-m/r}^{\infty}\phi(z)\,dz+r\int_{-m/r}^{\infty}z\phi(z)\,dz=m\left(1-\Phi\left(-m/r\right)\right)+r\phi\left(-m/r\right)$$

$$=m\Phi\left(m/r\right)+r\phi\left(m/r\right),$$

VOI for Gaussian

Result:

Gaussian cdf

Gaussian pdf

$$VOI(x) = m\Phi\left(m/r\right) + r\phi\left(m/r\right) - \max\{0, m\}$$

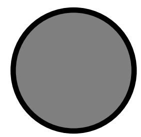
The analytical form facilitates computing, and eases the study of VOI properties as a function of the parameters.

$$m = 0,$$

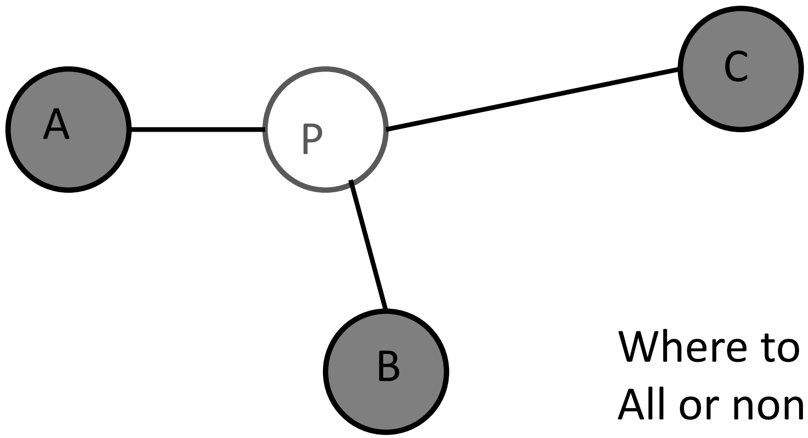
$$VOI(x) = r\phi(0) = \frac{r}{\sqrt{2\pi}}$$

The more uncertain, the more valuable is information.

What if several projects or treasures?



What if several projects or treasures?



Where to invest?
All or none? Free to choose as many as profitable? One at a time, then choose again?

Where should one collect data? All or none? One only? Or two? One first, then maybe another?

VOI and Earth sciences

- **Alternatives are spatial**, often with high flexibiliy in selection of sites, control rates, intervention, excavation opportunities, harvesting, etc.

- **Uncertainties are spatial**, with multi-variable interactions . Often both discrete and continuous.

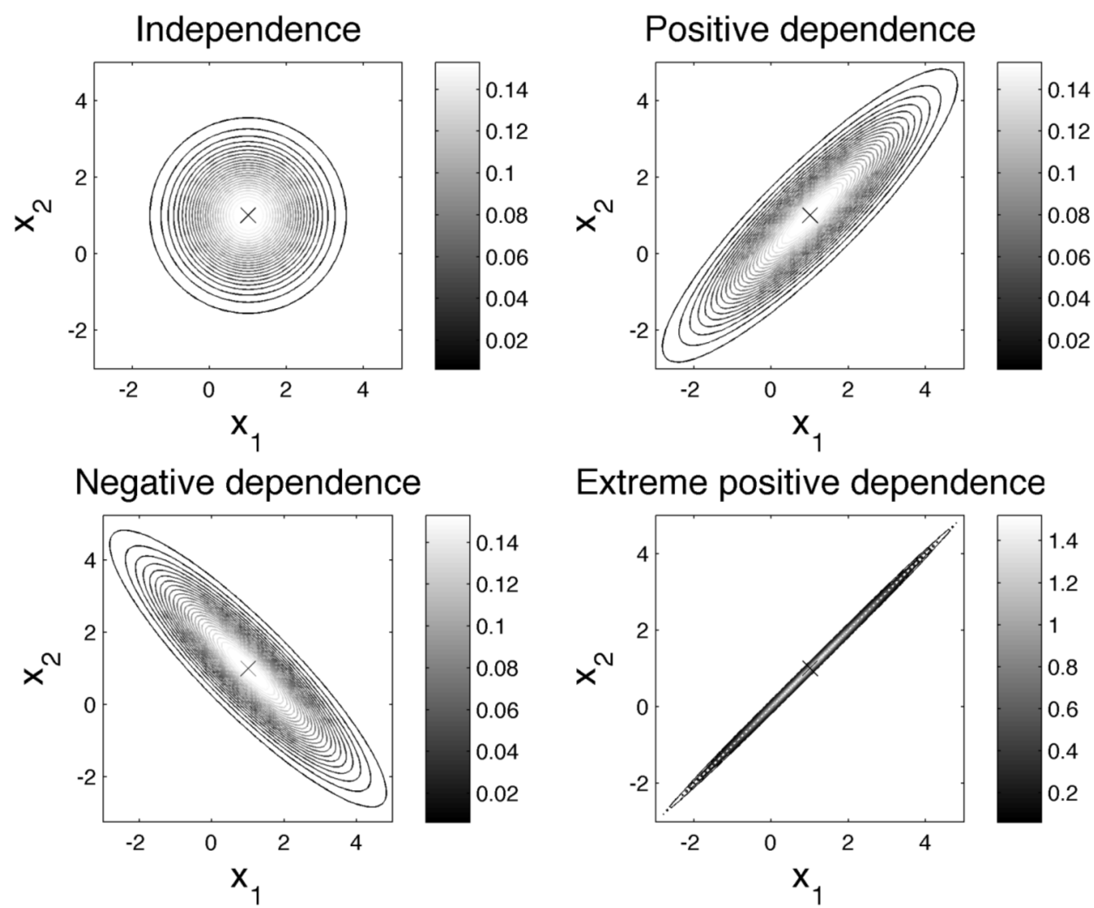
- **Value function is spatial**, typically involving coupled features, say through differential equations. It can be defined by «physics» as well as economic attributes.

- **Data are spatial**. There are plenty opportunities for partial, total testing and a variety of tests (surveys, monitoring sensors, electromagnetic data, , etc.)

Two-project example

Two correlated projects with uncertain profits.

Decision maker considers investing in project(s).



Gaussian projects example

- **Alternatives**
  - Do not invest in project 1 (a1=0) - Invest in project 1 (a1=1)
  - Do not invest in project 2 (a2=0) - Invest in project 1 (a2=1)
  - Decision maker is free to select both, if profitable: Four sets of alternatives.
- **Uncertainty** (random variable)
  - Profits are <u>*bivariate Gaussian*</u>.
    Assume mean 0, variance 1 and fixed correlation.
- **Value** decouples to sum of profits, if positive.
- **Information gathering**
  - Report can be written about one project (assume perfect).
  - Report can be written about both projects (assume imperfect).

Gaussian projects example

$$\boldsymbol{x} = (x_1, x_2)$$

Prior model for profits: $p(\boldsymbol{x}) = N(\boldsymbol{0}, \boldsymbol{\Sigma}),\qquad \boldsymbol{\Sigma} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$

$$PV = \sum_{i=1}^{2} \max\left\{0, E(x_i)\right\} = 0 + 0 = 0$$

$$PoV(\boldsymbol{y}) = \sum_{i=1}^{2} \int \max\left\{0, E(x_i \mid \boldsymbol{y})\right\} p(\boldsymbol{y}) \, d\boldsymbol{y}$$

$$\mathrm{VOI}(\boldsymbol{y}) = PoV(\boldsymbol{y}) - PV$$

Gaussian projects example

$$PV = \sum_{i=1}^{2} \max\{0, E(x_i)\} = 0 + 0 = 0$$

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$$\mathrm{VOI}(\boldsymbol{y}) = PoV(\boldsymbol{y}) - PV$$

Must solve the integral expression!

Need conditonal expectation!

Need marginal for data!

Perfect information about 1 project

$$y = x_1$$
$$p(x_1) = N(0,1)$$
$$E(x_1) = x_1$$
$$E(x_2 \mid x_1) = \rho x_1$$

Get information about second project because of correlation!

$$Var(x_1 \mid x_1) = 0, Var(x_2 \mid x_1) = 1 - \rho^2$$
$$PoV(x_1) = \int_0^\infty x_1 p(x_1)\, dx_1 + \int_0^\infty |\rho| x_1 p(x_1)\, dx_1$$
$$= \left(1 + |\rho|\right) \Big/ \sqrt{2\pi}$$

Imperfect information, both projects

$$\boldsymbol{y} = \boldsymbol{x} + N\left(\boldsymbol{0}, \tau^2 \boldsymbol{I}\right)$$

$$p\left(\boldsymbol{y}\right) = N\left(\boldsymbol{0}, \tau^2 \boldsymbol{I} + \boldsymbol{\Sigma}\right) = N\left(\boldsymbol{0}, \boldsymbol{C}\right)$$

$$E\left(\boldsymbol{x} \mid \boldsymbol{y}\right) = \boldsymbol{\Sigma}\boldsymbol{C}^{-1}\boldsymbol{y}$$

Reduction in variances large, VOI is large.

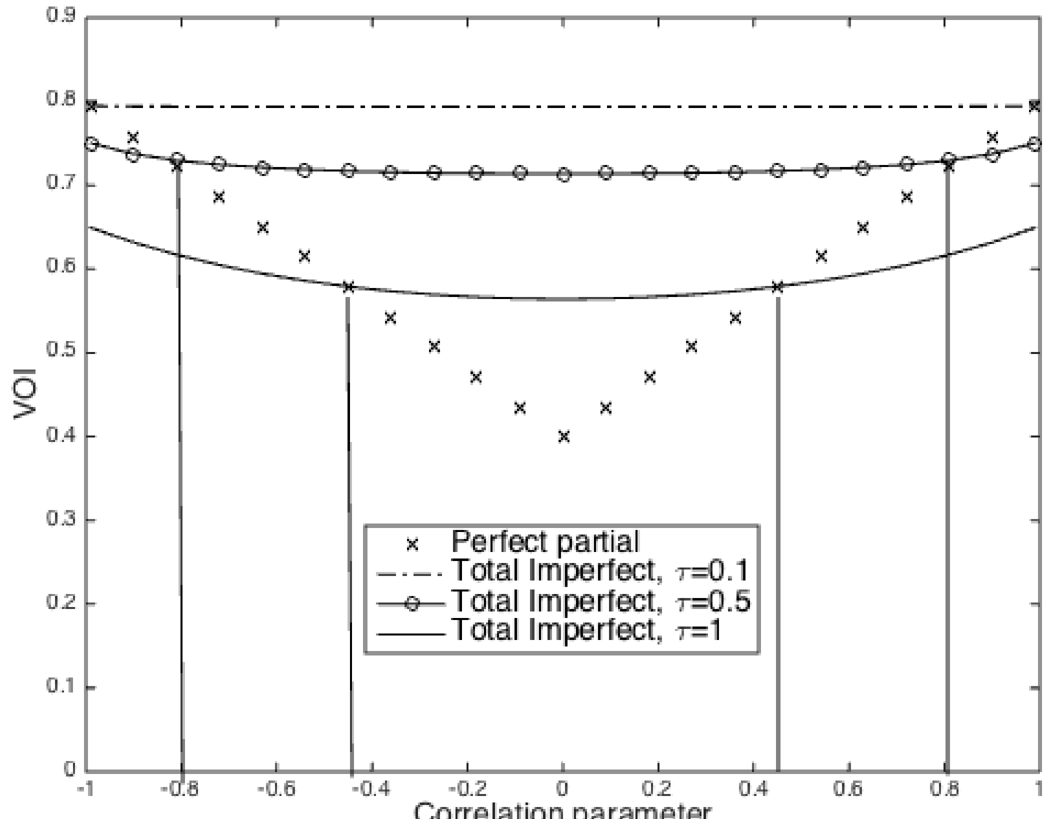
$$Var\left(\boldsymbol{x} \mid \boldsymbol{y}\right) = \boldsymbol{\Sigma} - \boldsymbol{R}, \qquad \boldsymbol{R} = \boldsymbol{\Sigma}\boldsymbol{C}^{-1}\boldsymbol{\Sigma}$$

$$PoV\left(\boldsymbol{y}\right) = \sum_{i=1}^{2} \int \max\left\{0, E\left(x_i \mid \boldsymbol{y}\right)\right\} p\left(\boldsymbol{y}\right) d\boldsymbol{y} = \left(\sqrt{R_{1,1}} + \sqrt{R_{2,2}}\right) \Big/ \sqrt{2\pi}$$

Gaussian projects results

$$PoV(\boldsymbol{y}) = \left(\sqrt{R_{1,1}} + \sqrt{R_{2,2}}\right) \Big/ \sqrt{2\pi}, \qquad \boldsymbol{R} = \boldsymbol{\Sigma}\boldsymbol{C}^{-1}\boldsymbol{\Sigma}$$

$$PoV(x_1) = \left(1 + |\rho|\right) \Big/ \sqrt{2\pi}$$

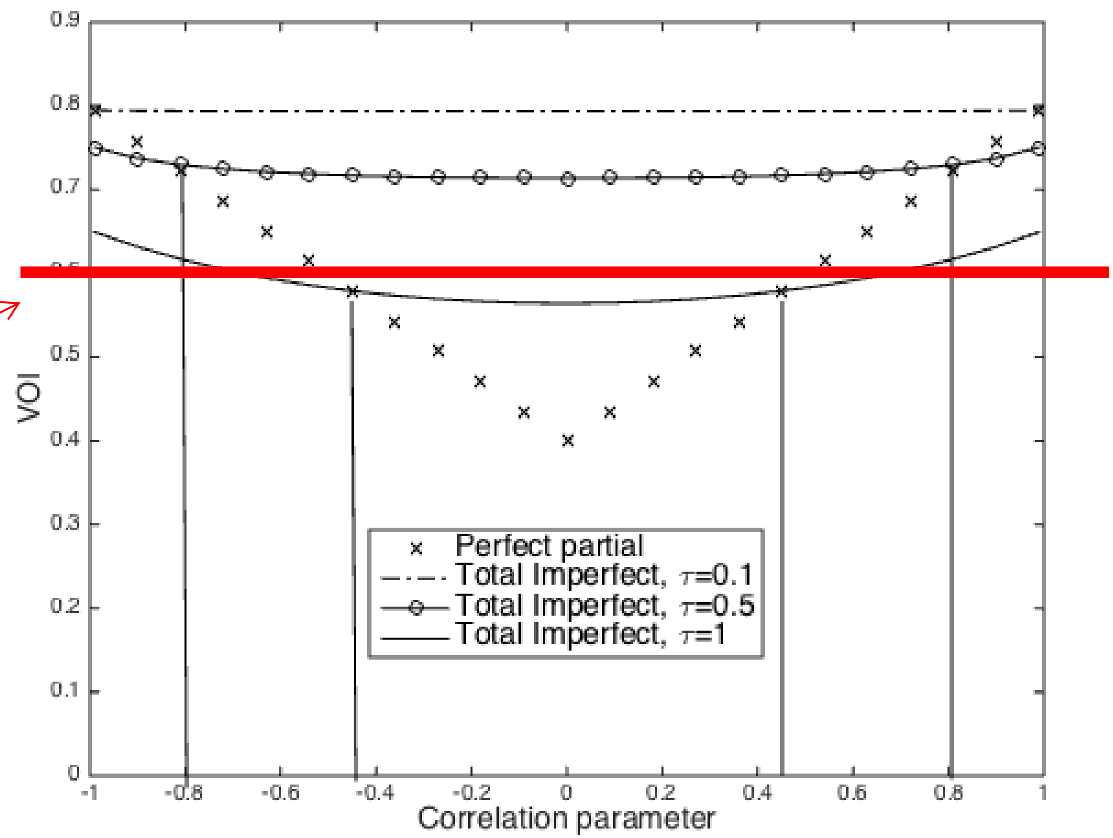


Gaussian projects results

$$PoV(\mathbf{y}) = \left(\sqrt{R_{1,1}} + \sqrt{R_{2,2}}\right) \Big/ \sqrt{2\pi}, \qquad \mathbf{R} = \mathbf{\Sigma C}^{-1}\mathbf{\Sigma}$$

$$PoV(x_1) = \left(1 + |\rho|\right) \Big/ \sqrt{2\pi}$$

Price of test.



Insight from Gaussian projects

Dependence matters – the more correlation, the larger VOI.
The relative increase is very clear for partial information. It is also larger when there is more measurement noise. (With perfect total information, dependence does not matter.)

Decision maker must compare the VOI with the price of information, and purchase the data if the VOI exceeds the price.

Plan for course

Time	**Topic**
Monday	Introduction and motivating examples
	Elementary decision analysis and the value of information
Tuesday	Multivariate statistical modeling, dependence, graphs
	Value of information analysis for dependent models
Wednesday	Spatial statistics, spatial design of experiments
	Value of information analysis in spatial decision situations
Thursday	Examples of value of information analysis in Earth sciences
	Computational aspects
Friday	Sequential decisions and sequential information gathering
	Examples from mining and oceanography

Every day: Exercises half-way, and computer project at the end.

Project 1 : Gaussian projects

Implement the bivariate Gaussian projects example, with prior mean 0 and variance 1, correlation parameter and measurement noise st dev parameter.

- Compute and plot the VOI for different correlation parameters (0.01-0.99) and a couple of st dev parameters (0.01-0.50)
- Study the *decision regions* for no testing, partial (1 only) or total imperfect testing.

Decision regions are useful for comparing the VOI results of ‘no testing’, ‘partial’ or ‘total’ tests, with the price P1 of first test, and P2 of second test:

$$\arg\max\left\{VOI_{1,2}-\left(P_1+P_2\right),VOI_1-P_1,0\right\}$$

Use, say, correlation 0.7, measurement st dev 0.25, and prices (0.01-1) for P1 and P2.